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Revisiting wave friction factors for rough turbulent flow

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Abstract

The wave friction factor f_w and the phase lead of the seabed shear stress over the free stream velocity φ for rough turbulent flow are revisited by utilizing the similarity theory used by Myrhaug (1989). Results are obtained for f_w and φ by determining the similarity coefficients using the Dixen et al. (2008) data for large bed roughness. Comparisons are made with other experimental data and wave friction factor formulae. As a result, an approximation for f_w by disregarding the phase φ is recommended covering a wide range of amplitude-to-roughness ratios.

Keywords

Seabed shear stress; Rough turbulent flow; Ocean surface waves; Coastal zones

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1. Introduction

Near-coastal zones are generally characterized by their intermediate and shallow water depths where the flow is caused by ocean surface gravity waves and currents. Coastal flow circulation models represent tools commonly used in near-coastal work, and these models often apply parameterizations of many flow mechanisms, such as dissipation mechanisms of the waves, for example, the seabed shear stress which is caused by the boundary layer-related friction between the fluid and the seabed. Knowledge of the seabed shear stress is also important since it affects the sediment transport and morphology and thus the stability of scour protections in coastal zones. More details on the background together with a review of the topic are found in the recent textbook of Sumer and Fuhrman (2020).

The purpose of this article is to revisit the seabed shear stress beneath gravity waves which commonly is expressed in terms of the wave friction factor. Here the wave friction factor f_w and the phase lead of the seabed shear stress over the free stream wave-induced velocity φ for rough turbulent flow is revisited by utilizing the similarity theory used by Myrhaug (1989) (hereafter referred to as M89). Results are obtained for f_w and φ by determining the coefficients as required by similarity theory using the low amplitude to roughness ratio data from Dixen et al. (2008). Comparisons are also made with other experimental data (Bagnold, 1946; Jensen et al., 1989; Jonsson and Carlsen, 1976; Kamphuis, 1975; Myrhaug et al., 1988; Simons et al., 2000; Sleath, 1987; Sumer et al., 1987) as well as with other wave friction factor formulae (Fredsøe and Deigaard, 1992; Fuhrman et al., 2013; Kamphuis, 1975; Nielsen, 1992; Simons et al., 2000; Sleath, 1991; Soulsby, 1997; Swart, 1974). As a result, an approximation for f_w by disregarding the phase φ is recommended for the amplitude-to-roughness ratio larger than about 0.2. Thus, the present formula should represent a useful parameterization of f_w since it covers a wide range of rough turbulent flow conditions.

The article is organized as follows. This introduction is followed by determining the similarity coefficients from the Dixen et al. (2008) data. Then, comparisons with other data and friction factor formulae are made. Finally, a summary and conclusions are given. Two appendices are also included by providing summaries of the similarity theory (Appendix A) and the friction factor formulae (Appendix B).

2. Similarity coefficients determined from the Dixen et al. (2008) data

The similarity theory used by M89 is summarized in Appendix A. Here the coefficients A and B in Eq. (A1) are determined according to Eqs. (A4) and (A5) by using the five data sets from measurements of the turbulent flow structure in rough turbulent wave boundary layers reported by Dixen et al. (2008). The main parameters and results from the experiments are summarized in Table 1, where Tests P1 to P5 refer to the results from wave flume experiments corresponding to the crest half period using ping-pong balls as roughness elements. In Table 1 $|u_0|$ is the magni-

Table 1. Main parameters and results from Dixen et al. (2008) for the ping-pong-ball experiments corresponding to the crest half period.

Test	$\lvert u_0 \rvert$ [m s^{-1}]	a_0 [m]	ω [rad s^{-1}]	k [mm]	u_* [m s^{-1}]	f_w	$\mathfrak{R}^*$	$\frac{a_0}{k}$	φ [deg]
P1	0.277	0.060	3.927	91	0.130	0.44	1.6×10^4	0.66	20
P2	0.279	0.061	3.927	90	0.136	0.48	1.6×10^4	0.68	19
P3	0.309	0.068	3.927	91	0.140	0.41	1.7×10^4	0.74	12
P4	0.325	0.072	3.927	90	0.138	0.36	2.2×10^4	0.79	16
P5	0.333	0.073	3.927	91	0.134	0.33	2.3×10^4	0.81	17

$^*\ v = 1.0646 \times 10^2$ cm^2 s^{-1} at water temperature 17.5–18.5°

Table 2. Coefficients determined from similarity theory using the Dixen et al. (2008) data (see Table 1).

Test	A	B	c	$\acute{c}$
P1	1.43	0.29	0.24	0.25
P2	1.53	0.27	0.22	0.23
P3	1.45	0.18	0.23	0.24
P4	1.44	0.26	0.24	0.25
P5	1.34	0.29	0.26	0.27

tude of the horizontal wave- induced velocity amplitude u_0, a_0 is the horizontal wave-induced fluid particle amplitude, ω is the angular wave frequency, k is the Nikuradse equivalent sand roughness, u_* is the friction velocity, f_w is the friction factor, $\mathfrak{R} = |u_0| \frac{a_0}{v}$ is the Reynolds number, v is the kinematic fluid viscosity, and φ is the phase lead of the seabed shear stress over u_0. More details are provided in Dixen et al. (2008).

The values of A and B and the corresponding values of $c = \exp(-A)$ in Eq. (A3) for Tests P1 to P5 are given in Table 2, together with the values of $\acute{c}$ determined from Eq. (A7), corresponding to using Eq. (A3) with B and $c = \acute{c}$. This is an approximation where the phase of the velocity within the wave boundary layer is disregarded, and the logarithmic boundary layer flow model in Eq. (A8) is extended beyond its range of validity (see Appendix A).

Here the average values of the coefficients in Table 2 are used, i.e.:

$$\overline{A} = 1.44, \quad \overline{B} = 0.26, \quad \bar{c} = 0.24, \quad \bar{\acute{c}} = 0.25 \tag{1}$$

Then, the friction factor f_w and the phase φ are determined from Eqs. (A3) and (A5), respectively, as:

$$f_w = \frac{2\kappa^2}{\left[\ln\left(30\bar{c}\,\frac{a_0}{k}\sqrt{\frac{f_w}{2}}\right)\right]^2 + \overline{B}^2} \tag{2}$$

$$\varphi = \arcsin\left(\frac{\overline{B}}{\kappa}\sqrt{\frac{f_w}{2}}\right) \tag{3}$$

When the phase is disregarded, f_w is determined from Eq. (2) with $\overline{B} = 0$ and $\bar{c} = \bar{\acute{c}}$.

3. Comparisons with data and friction factor formulae

The data and friction factor formulae included here cover a wide range of rough bed conditions, i.e. $0.2 < \frac{a_0}{k} < 4000$. First, and generally, it should be noted that the total horizontal force acting on the bed roughness consists of the shear stress on the bed plus the components caused by the mean pressure gradient acting on the exposed surface of the bed roughness. Sleath (1991) gave the following friction factor formula for sinusoidal waves taking account of the total horizontal force acting on the exposed surface of grains of sediments tightly packed in a single layer on a flat plate for data in the range $1 < a_0/k < 120$:

$$f_w = (D^2 + E^2 + 2DE\sin\theta)^{\frac{1}{2}} \tag{4}$$

where $\theta = 22.5°$, and the two terms

$$D = 0.048\left(\frac{a_0}{k}\right)^{-0.25} \tag{5}$$

$$E = 0.60\left(\frac{a_0}{k}\right)^{-1} \tag{6}$$

represent the components due to the shear stress and the mean pressure gradient, respectively. For small and large values of $\frac{a_0}{k}$ the terms E and D, respectively, dominate in Eq. (4). Thus, for the lower and higher values of $\frac{a_0}{k}$ good approximations of Eq. (4) are Eqs. (6) and (5), respectively, i.e.:

$$f_w = 0.60\left(\frac{a_0}{k}\right)^{-1} \tag{7}$$

$$f_w = 0.048\left(\frac{a_0}{k}\right)^{-0.25} \tag{8}$$

Next, the data and the friction factor formulae used for comparison are briefly described. Bagnold (1946) performed oscillating plate tests artificially roughened with ripples. The friction factor for small $\frac{a_0}{k}$ in the rough turbulent flow regime were obtained by measuring the drag on the plate. Kamphuis (1975) determined the friction factor measuring the shear stress in oscillating water tunnel

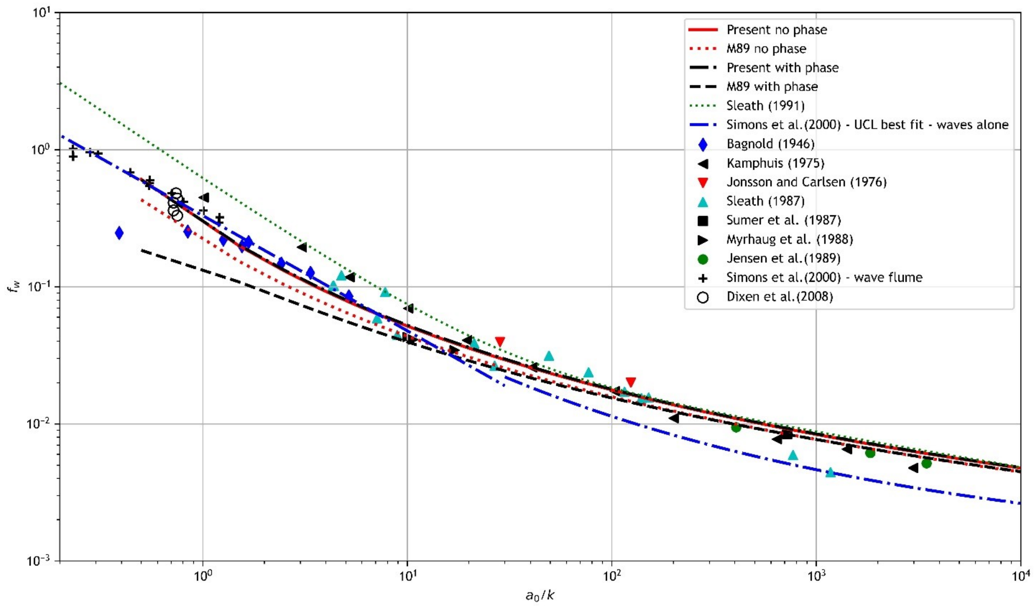

Figure 1. f_w versus $\frac{a_0}{k}$ for rough turbulent flow.

tests on both smooth and sand-roughened plates, covering a wide range of flow regimes for laminar flow, transitional laminar to turbulent flow and turbulent flow. Kamphuis (1975) proposed the friction factor formula in Eq. (B3) (Appendix B) obtained as fit to his data for $10 < \frac{a_0}{k} < 50$. Jonsson and Carlsen (1976) did oscillating water tunnel tests and determined the friction factors from velocity measurements under rough turbulent flow conditions. Sleath (1987) deduced friction factors from both the Reynolds shear stress and the momentum integral equation (presented here) based on measurements of velocities over rough beds. Sumer et al. (1987) carried out experiments in an oscillating water tunnel (in the same tunnel as Jonsson and Carlsen (1976)) measuring the turbulent flow structure near smooth and rough beds. The rough bed was covered with fairly uniform sized sand. The streamwise and transverse velocity components were measured using a one-component Laser Doppler Anemometer (LDA), and u_* and k were determined by fitting straight lines to the logarithmic layer part of the velocity data. In addition, u_* was determined from the momentum integral equation using the experimental velocity profiles. The values of u_* determined in both ways agreed well. This work was extended by Jensen et al. (1989) by performing an extensive set of measurements on smooth, transitional smooth to rough, and rough turbulent wave boundary layer flow. Sumer et al. (1987) and Jensen et al. (1989) provided results of the mean and turbulent flow structure including measurements of velocity profiles, bed shear stresses and the root-mean-square (rms) values of the horizontal turbulent fluctuations. Here the rough bed results are used. Myrhaug et al. (1988) presented results from a large-scale laboratory experiment on wave boundary layer flow over an artificially rough bed using three-dimensional roughness elements with a physical height of 4 cm. The data used here were obtained from the damped oscillator model fit to the data (Myrhaug et al., 1992). Simons et al. (2000) carried out laboratory experiments studying the bed friction in waves alone and in combined wave-current flow. The results included measurements of the shear stress for both regular and irregular (bi-chromatic and bi-directional) waves with a superimposed current. Direct measurements of the bed shear stress were made using a shear cell, and the velocity profiles were recorded simultaneously using three acoustic Doppler velocimeters. The present results include the bed friction for regular waves alone. The friction coefficient formulae in Eqs. (B7) and (B8) were obtained as best fit to data. Dixen et al. (2008) summarized results of a detailed experimental investigation on wave boundary layer over a bed with large roughness using stones as well as ping-pong balls, where the results for the ping-pong balls are used here. The tests were carried out in wave flumes and the bed friction was determined both from the log-fit method, the momentum integral method using the measured velocity profiles as well as from the Reynolds shear stress measurements. The ping-pong balls results are in the range $0.66 < \frac{a_0}{k} < 0.81$ (Table 1). For very rough beds they also discussed the details of the flow in close vicinity to the roughness elements where the major contribution to the friction factor is from the mean pressure gradient consistent with the results of Sleath (1991). For $0.2 < \frac{a_0}{k} < 4$ they proposed the friction factor formula in Eq. (B9).

In addition to the friction factor formulae referred to,

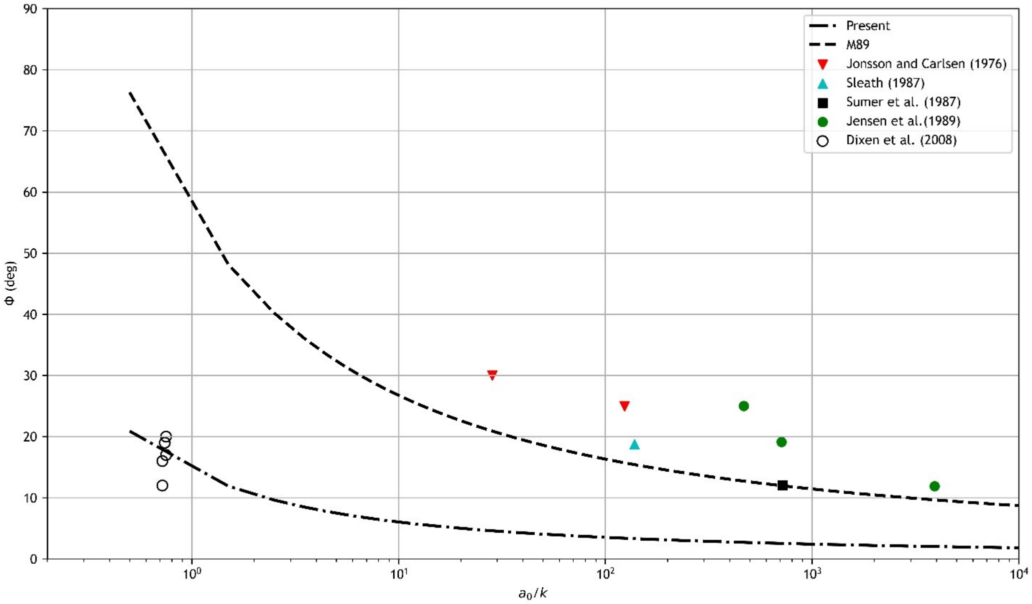

Figure 2. φ versus $\frac{a_0}{k}$ for rough turbulent flow.

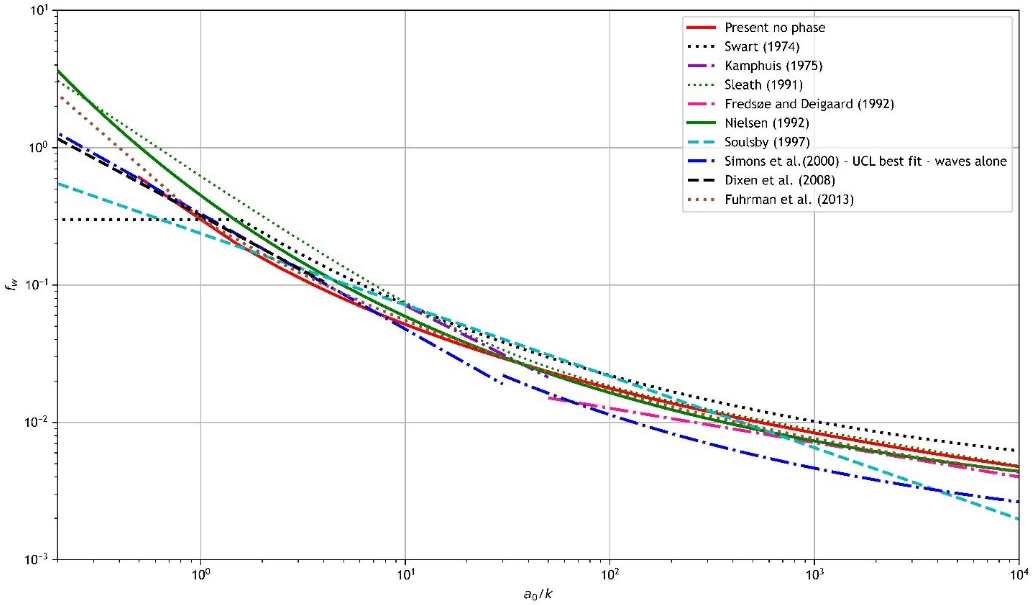

Figure 3. f_w versus $\frac{a_0}{k}$ for rough turbulent flow.

some other formulae frequently applied in the literature are included for comparisons here: Fredsøe and Deigaard (1992) (Eq. (B5)), Fuhrman et al. (2013) (Eq. (B10)), Nielsen (1992) (Eq. (B4)), Soulsby (1997) (Eq. (B6)), Swart (1974) (Eqs. (B1) and (B2)). More details of the data and the friction factor formulae referred to here are found in the respective references.

Figures 1 and 2 show the friction factor f_w (Figure 1) and the phase φ (Figure 2) versus the amplitude to roughness ratio $\frac{a_0}{k}$. Figure 1 depicts the rough turbulent flow data from Bagnold (1946), Dixen et al. (2008), Jensen et al. (1989), Jonsson and Carlsen (1976), Kamphuis (1975), Myrhaug et al. (1988), Simons et al. (2000), Sleath (1987), Sumer et al. (1987), together with the present predictions according to Eq. (2) with $\overline{B}=0.26,\ \overline{c}=0.24$ (including the phase), and Eq. (2) with $\overline{B}=0,\ \overline{c}=\overline{\hat{c}}=0.25$ (neglecting the phase). The figure also includes the M89 results according to Eq. (2) with $\overline{B}=1.28,\ \overline{c}=0.30$ (including the phase), and Eq. (2) with $\overline{B}=0,\ \overline{c}=\overline{\hat{c}}=0.34$ (neglecting the phase), the Simons et al. (2000) formula obtained as best fit to waves alone data (Eqs. (B7) and (B8)), as well as the Sleath (1991) formula in Eqs. (4)–(6). Rather than plotting all the Simons et al. (2000) data points, the formula is taken to represent the data. Figure 2 depicts the data from Dixen et al. (2008), Jonsson and Carlsen (1976), Sleath (1987), Sumer et al. (1987), the present predictions according to Eqs. (3) and (2) with $\overline{B} = 0.26$, $\overline{c} = 0.24$, and the M89 results according to Eqs. (3) and (2) with $\overline{B} = 1.28$, $\overline{c} = 0.30$.

From Figure 1 it appears that the overall agreement between the present semi-empirical predictions and the experimental data is reasonable except for the Bagnold data for $\frac{a_0}{k} < 1$, the Kamphuis (1975) data for $\frac{a_0}{k} < 10$, the Simons et al. (2000) data for $\frac{a_0}{k}$ larger than about 20 (note that the Simons et al. (2000) formula represents the data), and the Sleath (1987) data for some of the data for $\frac{a_0}{k} < 10$ and $\frac{a_0}{k}$ around 10^3. It should be noted that the present semi-empirical predictions of f_w both by including and disregarding the phase yield nearly overlapping results over the wide $\frac{a_0}{k}$ range. It is also noted that the present predictions deviate from the M89 predictions based on the fit to the Sumer et al. (1987) data (for $\frac{a_0}{k} = 720$) as $\frac{a_0}{k}$ decreases; the deviations are larger for the M89 predictions disregarding the phase. The Sleath (1991) formula agrees well with the Kamphuis (1975) data for $\frac{a_0}{k} > 3$, the Sleath (1987) data except for those with $\frac{a_0}{k}$ around 10^3, and it also agrees well with the other data for $\frac{a_0}{k} > 30$, except for the Simons et al. (2000) data/formula. For $\frac{a_0}{k} < 30$ the other data is overpredicted.

From Figure 2 it appears that there are distinct differences between the present predictions of the phase and those by M89 based on the fit to the $\frac{a_0}{k}$ Sumer et al. (1987) data. The data from Jensen et al. (1989), Jonsson and Carlsen (1976), and Sleath (1987) are also included. The values are in the range 12° to 30° demonstrating a large scatter, but overall the values do not change significantly within the large $\frac{a_0}{k}$ range. As explained by Dixen et al. (2008): ”This mechanism is closely related to the process where the lee-wake water is washed over the roughness elements prior to the flow reversal in the potential-flow region. Since this mechanism does not change with changing the roughness parameter, the phase lead φ will therefore remain practically unchanged”. Thus, it appears that the similarity theory is not capable of predicting φ over the large $\frac{a_0}{k}$ range.

Figure 3 depicts a replot of Figure 1 including the present formula disregarding the phase plus the friction factors referred to earlier, but the data are not shown here. It appears that the present predictions disregarding the phase agree well with Dixen et al. (2008), Fredsøe and Deigaard (1992) for $\frac{a_0}{k} > 10^3$, Fuhrman et al. (2013) for $\frac{a_0}{k} > 0.5$, Kamphuis (1975), Nielsen (1992) for $\frac{a_0}{k} > 1$, Simons et al. (2000) except for larger than about 20, Sleath (1991) for $\frac{a_0}{k} > 30$, Soulsby (1997) for $1 < \frac{a_0}{k} < 10^3$, and Swart (1974) for $\frac{a_0}{k} > 1.57$. It is noticed that the Fredsøe and Deigaard (1992) formula (Eq. (B5)) differs slightly from the large $\frac{a_0}{k}$ behaviour of the Sleath (1991) formula in Eq. (8), i.e. 0.048 in Eq. (8) is slightly larger than 0.04 in Eq. (B5). It is also noticed that the Sleath (1991) formula for lower $\frac{a_0}{k}$ in Eq. (7) is larger than the other formulae, except for the Nielsen (1992) formula; the two formulae are close for lower $\frac{a_0}{k}$.

Based on the present comparison with the data and the friction factor formulae and the reasonable agreement with some of the data and the formulae it is recommended to use the approximation for f_w by disregarding the phase φ, i.e. Eq. (2) with $\overline{B} = 0$ and $\overline{c} = 0.25$, which is strictly valid for $\frac{a_0}{k} \geq 0.66$. The reason is that this is a result of fitting Eq. (2) to the Dixen et al. (2008) data valid for $0.66 \leq \frac{a_0}{k} \leq 0.81$ (see Table 1). However, Figures 1 and 3 indicate that the formula can be extended to be valid for $\frac{a_0}{k} > 0.2$.

4. Summary and conclusions

The wave friction factor f_w and the phase lead φ of the seabed shear stress over the free stream wave-induced velocity for rough turbulent flow are provided by utilizing the similarity theory used by Myrhaug (1989). Here the Dixen et al. (2008) data representing low amplitude to roughness ratios $\frac{a_0}{k}$ are used to determine the coefficients as required by similarity theory. Comparisons are made with other data as well as other wave friction factor formulae covering a wide range of $\frac{a_0}{k}$. Based on these comparisons and the reasonable agreement with some of the data and the formulae it is recommended to use the present approximation of f_w by disregarding the phase φ for $\frac{a_0}{k} > 0.2$. Thus, the present formula for f_w should represent an alternative useful parameterization since it covers a wide range of $\frac{a_0}{k}$ for rough turbulent flow conditions.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Similarity theory

Here a summary of the similarity theory used by M89 is provided, utilizing the well-known analogy between wave boundary layer flow and planetary boundary layer flow (Grant and Madsen, 1986; Soulsby, 1983). According to similarity theory and following Gill (1982, Ch. 9.5) the horizontal wave-induced velocity amplitude u_0 associated with the oscillatory wave motion outside the wave boundary layer can be written as

$$u_0 = \frac{u_*}{\kappa}\left(\ln\frac{u_*}{\omega z_0} - A - iB\right) \tag{A1}$$

Here $u_0 = u_{0r} + iu_{0i}$ is a complex representation with u_{0r} and u_{0i} as real and imaginary parts, respectively, $u_* = \left(\frac{\tau_0}{\rho}\right)^{1/2}$ is the friction velocity, ρ is the fluid density, τ_0 is the maximum shear stress at the seabed which has a phase lead φ over u_0, $\kappa = 0.4$ is the von Karman constant, z_0 is the seabed roughness parameter, ω is the angular wave frequency, A and B are coefficients as required by similarity theory.

The wave friction factor is defined in terms of the maximum seabed shear stress as

$$f_w = \frac{\tau_0}{\frac{1}{2}\rho|u_0|^2} = 2\left(\frac{u_*}{|u_0|}\right)^2 \tag{A2}$$

where $|u_0| = \left(u_{0r}^2 + u_{0i}^2\right)^{1/2}$ is the magnitude of u_0, i.e. the wave-induced velocity amplitude or the maximum wave-induced velocity outside the wave boundary layer. By defining the horizontal wave-induced fluid particle amplitude $a_0 = \frac{|u_0|}{\omega}$ and using Eq. (A2), the magnitude of Eq. (A1) is

$$\frac{2\kappa^2}{f_w} = \left[\ln\left(c\frac{a_0}{z_0}\sqrt{\frac{f_w}{2}}\right)\right]^2 + B^2 \tag{A3}$$

where $c = \exp(-A)$ is a coefficient. Now $u_{0r} = |u_0|\cos\varphi$ and $u_{0i} = -|u_0|\sin\varphi$, which expresses the phase lead of the maximum seabed shear stress over the maximum wave-induced velocity outside the wave boundary layer. By using this and Eq. (A2), A and B are given from Eq. (A1) as

$$A = \ln\left(\frac{a_0}{z_0}\sqrt{\frac{f_w}{2}}\right) - \frac{\kappa}{\sqrt{\frac{f_w}{2}}}\cos\varphi \tag{A4}$$

$$B = \frac{\kappa}{\sqrt{\frac{f_w}{2}}}\sin\varphi \tag{A5}$$

which determine A and B for a given data set. Then, f_w is given from Eq. (A3) and φ is obtained from Eq. (A5) as

$$\varphi = arc\sin\left(\frac{B}{\kappa}\sqrt{\frac{f_w}{2}}\right) \tag{A6}$$

An approximation to Eq. (A3) is

$$\frac{\kappa}{\sqrt{\frac{f_w}{2}}} = \ln\left(\acute{c}\frac{a_0}{z_0}\sqrt{\frac{f_w}{2}}\right) \tag{A7}$$

or, corresponding equivalently to

$$|u_0| = \frac{u_*}{\kappa}\ln\left(\frac{\acute{c}u_*}{\omega z_0}\right) \tag{A8}$$

This approximation implies that the phase of the velocity within the boundary layer is disregarded, and consequently that the logarithmic boundary layer flow model (Eq. (A8)) is extended beyond its validity range. The logarithmic boundary layer flow model is only valid within a region where the shear stress is constant, while here it is extended to the height $\frac{\acute{c}u_*}{\omega}$ where the velocity equals the maximum wave-induced velocity outside the boundary layer. Thus, the coefficient $\acute{c}$ is obtained from Eq. (A8) for a given data set.

The roughness parameter for a rough seabed is

$$z_0 = \frac{k}{30} \tag{A9}$$

where k is the Nikuradse equivalent sand roughness corresponding to the characteristic dimension of the physical roughness of the seabed although k may be very different from what the physical roughness of the surface would suggest (Schlichting, 1979; Ch. 20). For a smooth surface the roughness parameter is

$$z_0 = \frac{v}{9u_*} \tag{A10}$$

where v is the kinematic fluid viscosity (Schlichting, 1979; Ch. 20). Thus, the velocity is taken to be zero at a fixed level z_0 above the surface rather than at $z = 0$. For transitional smooth-to-rough turbulent flow, the roughness parameter (Christoffersen and Jonsson, 1985) is

$$z_0 = \frac{k}{30}\left[1 - \exp\left(-\frac{1}{27}\frac{ku_*}{v}\right)\right] + \frac{v}{9u_*} \tag{A11}$$

which is obtained as a fit to the data points in Schlichting (1979, Fig. 20.21). For $v = 0$ and $k = 0$, Eq. (A11) reduces to Eqs. (A9) and (A10), respectively.

The following turbulent flow regimes are given by Schlichting (1979; Ch. 20)

$$\text{smooth}: \quad 0 < \frac{ku_*}{v} < 5 \tag{A12}$$

$$\text{transition}: \quad 5 < \frac{ku_*}{v} < 70 \tag{A13}$$

$$\text{rough :} \qquad 70 < \frac{ku_*}{v} \tag{A14}$$

More details on the results for smooth turbulent and transitional smooth-to-rough turbulent flow are provided in M89.

Appendix B. Friction factor formulae

The friction factor formulae referred to in the main text is summarized here.

Swart (1974)

$$f_w = 0.3; \ \frac{a_0}{k} < 1.57 \tag{B1}$$

$$f_w = 0.00251 \exp\left[5.21\left(\frac{a_0}{k}\right)^{-0.19}\right]; \ \frac{a_0}{k} > 1.57 \tag{B2}$$

Kamphuis (1975)

$$f_w = 0.4\left(\frac{a_0}{k}\right)^{-0.75} ; \quad 10 < \frac{a_0}{k} < 50 \tag{B3}$$

Nielsen (1992)

$$f_w = \exp\left[5.5\left(\frac{a_0}{k}\right)^{-0.2} - 6.3\right]; \quad \text{for all } \frac{a_0}{k} \tag{B4}$$

Fredsøe and Deigaard (1992)

$$f_w = 0.04\left(\frac{a_0}{k}\right)^{-0.25} ; \ \frac{a_0}{k} > 50 \tag{B5}$$

Soulsby (1997)

$$f_w = 0.237\left(\frac{a_0}{k}\right)^{-0.52} ; \text{ for all } \frac{a_0}{k} \tag{B6}$$

Simons et al. (2000)

$$f_w = 0.33\left(\frac{a_0}{k}\right)^{-0.84} ; \ \frac{a_0}{k} < 30 \tag{B7}$$

$$f_w = 0.001 \exp\left[6.1\left(\frac{a_0}{k}\right)^{-0.2}\right]; \ \frac{a_0}{k} > 30 \tag{B8}$$

Dixen et al. (2008)

$$f_w = 0.32\left(\frac{a_0}{k}\right)^{-0.8} ; 0.2 < \frac{a_0}{k} < 4 \tag{B9}$$

Fuhrman et al. (2013)

$$f_w = \exp\left[5.5\left(\frac{a_0}{k}\right)^{-0.16} - 6.7\right]; \quad \text{for all } \frac{a_0}{k} \tag{B10}$$