

Seabed saturation conditions from in-situ measurements performed on Norderney

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Abstract

This paper deals with an important coastal engineering problem of defining proper seabed saturation conditions, which have a significant influence on the pore-fluid compressibility and the wave-induced cyclic response of poro-elastic seabed sediments. A unique in-situ measuring campaign was conducted in the tidal zone of the northern beach of Norderney, off the North Sea coast of Germany, where 186 sandy seabed samples were taken underwater. Based on the laboratory measurements, a set of calculated saturation degrees was statistically analysed. Both the histogram and the normal Q-Q plot, as well as the Shapiro-Wilk normality test, confirmed the validity of the assumption of the normal probability distribution for the variability of the degree of saturation. The mean degree of saturation of the top layer of the seabed, $\bar{S}_r = 0.973$, constitutes the main output of the study, whereas the uncertainty propagation analysis enabled to define the possible range of variation, which is $0.962 \leq \bar{S}_r \leq 0.986$. It should be clearly emphasised that a proper assessment of the seabed saturation conditions is very important, mainly due to the correctness of the description of the wave-induced pore-fluid pressure field used in more detailed analyses of the pore-fluid pressure gradients and the liquefaction potential of the seabed, which have a direct impact on phenomena such as sand transport on beaches, seabed erosion, and stability of coastal structures (e.g. breakwaters and submarine buried pipelines).

Keywords

Coastal engineering; Intertidal zone; Partly saturated seabed; Degree of saturation; In-situ seabed sampling; Statistical analysis

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1. Introduction

Many more advanced solutions for the prediction of the wave-induced cyclic seabed response (by means of the pore-fluid pressure and stresses in the soil matrix) are strongly dependent on the compressibility of the two-phase (pore-fluid and soil skeleton) seabed medium. The pore-fluid is treated as a mixture of seawater and air entrapped in pores of seabed sediments in the form of occluded micro air bubbles. The relationship between the amount of free air in the pore-fluid, represented by the degree of saturation, and the compressibility of the pore-fluid can be given by a common and very practical formula proposed by Verrijt (1969). This relationship has been used for decades by many researchers, among others: Madsen (1978), Yamamoto et al. (1978), Nago and Maeno (1984), Okusa (1985), Magda (1994, 1998), Hsu and Jeng (1994), Jeng (2013, 2018), indicating a very strong dependence of the

wave-induced pore-fluid cyclic response on seabed saturation conditions. The degree of saturation plays a very important role in the process of pore-fluid pressure generation, both cyclic and residual, within seabed sediments. Therefore, this geotechnical parameter has a great bearing on many applications in coastal and offshore engineering. Any changes in the degree of saturation, even by a relatively small increment, in the range that is supposed to be found under natural conditions, mainly due to a certain inhomogeneity of seabed sediments, cause significant changes in the pore-fluid compressibility, particularly when the degree of saturation is close to unity (almost fully saturated seabed conditions).

What is the real value of the degree of saturation characterising seabed sediments? It is a difficult question remaining open. Inhomogeneity of seabed sediments and difficulties with very precise seabed sampling make the problem even worse. Many researchers (Table 1), when conducting analytical and numerical analyses of the wave-induced cyclic seabed response, prefer doing parametric

Table 1. A list (in chronological order) of some selected past research works using particular values of the degree of saturation, S_r , assumed in different analyses of the wave-induced seabed response.

Author/Authors	S_r [-]	Remarks
Madsen (1978)	0.965–1.0	Assumed in the parametric study.
Yamamoto et al. (1978)	0.95–1.0 0.98	Assumed in the parametric study. Obtained from the experiment by matching with the equation by Verruijt (1969).
Mei and Foda (1981)	0.92	Assumed as typical for the North Sea condition, recalculated from $G\beta = 10^4$ by Magda (2025).
Nago and Maeno (1984)	0.993	Assumed indirectly by taking the water and air porosities: $\lambda_w = 0.4$ and $\lambda_a = 0.003$, respectively.
Okusa (1985)	0.93–1.0 0.95–1.0	Assumed indirectly for loose sand by taking Skempton's pore-water coefficient $B = 0.5 - 1.0$. Assumed indirectly for dense sand by taking Skempton's pore-water coefficient $B = 0.5 - 1.0$.
Hsu and Jeng (1994)	0.966	Obtained by matching with the boundary-layer approximation by Mei and Foda (1981).
Ulker and Rahman (2009)	1.0	Assumed in the numerical experiments on the inertia effects.
Merxhani and Liang (2012)	1.0	Assumed in the analysis of the seabed response due to solitary waves.
Jeng (2013, 2018)	0.932	Obtained by matching with the boundary-layer approximation by Mei and Foda (1981).
Sumer (2014)	1.0 0.985	Assumed in the assessment of liquefaction potential. Assumed in the momentary liquefaction analysis.

38 studies with respect to the degree of saturation rather than
 39 performing demanding research for the degree of saturation
 40 typical for the case under consideration. This leads
 41 very often to presentations of the main computational results
 42 achieved for a wide range of possible values of S_r .
 43 Usually, this way of doing is fully justified and very desirable
 44 when presenting abilities of a newly derived analytical
 45 solution or looking for an optimum/maximum particular
 46 solution, as it is the case, for example, when the maximum
 47 uplift force acting on a submarine pipeline buried in seabed
 48 sediments is sought (Magda, 1992, 1997, 2000). It also
 49 happens that the researchers assume fully saturated soil
 50 conditions ($S_r = 1$) for the purpose of their analyses without
 51 giving any grounds for so doing. It is worth noting that
 52 only a few of the researchers reported some values of S_r ,
 53 measured in seabed sediments under natural conditions
 54 (Table 2).

55 Some other researchers try to match the results of their
 56 own computations with previously published results of

other authors by selecting a suitable value of the degree
 of saturation, which is a kind of back analysis. As typical
 examples thereof, the works by Hsu and Jeng (1994)
 and Jeng (2013, 2018) can be cited. Unfortunately, in this
 particular case, such action turned out to be disastrous,
 leading to questionable results. Hsu and Jeng (1994) and
 Jeng (2013, 2018) made many efforts in trying to replicate
 the seabed response results, obtained by means of
 pore-fluid pressure, soil displacements, and stresses, and
 published by Mei and Foda (1981) for the North Sea wave
 and seabed conditions, by applying their own analytical
 solutions. Unfortunately, they failed because multiple
 recalculations of one and the same case of the wave-induced
 seabed response required the usage of different values of
 the degree of saturation: $S_r = 0.966$ (Hsu and Jeng, 1994),
 $S_r = 0.932$ (Jeng, 2013, 2018), which, of course, cannot be
 acceptable. In addition, the author of the present paper
 made his own computations, using the analytical solution
 derived personally (Magda, 1994; Richwien and Magda,

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Table 2. A short list (in chronological order) of some selected past research works reporting particular values of the degree of saturation, S_r , based on in-situ measurements.

Author / Authors	S_r [-]	Remarks
Sandven et al. (2007)	0.988–1.0	Measured under falling (ebbing) tide conditions after repeated cycles of strong waves.
Tørum (2007)	0.97	Calculated by matching the field data with the boundary-layer theory by Mei and Foda (1981).
Michallet et al. (2009)	0.94–1.0	Calculated from the image analysis of the geoenoscopic video camera recording (minimum at a depth of 0.2–0.3 m).

1994), and the matching point was indicated for the degree of saturation to be equal to $S_r = 0.920$ (Magda, 2025). All together, there are three different values of S_r describing one and the same case! This particular bizarre example of extremely different values of the degree of saturation describing one and the same case can only emphasise the need for further investigations proving the real values of the degree of saturation under natural conditions. It seems that the authors of some research papers are freely juggling different values of the degree of saturation, having no well-established idea at what level the actual seabed saturation conditions may develop.

1.1 Objectives of the research

The main motivation of this research paper is to get a better insight into the problem of real saturation conditions by means of in-situ seabed sampling. A basic task of the research paper is to present the results of a unique measuring campaign performed on the northern beach of Norderney in order to gather samples of the upper seabed layer undergoing tidal movements. The results of laboratory measurements and additional calculations should give important statistics of the degree of saturation characterising natural environmental conditions that can be found in intertidal beach regions.

2. In-situ measuring campaign

The scientific expedition to Norderney, an island off the North Sea coast of Germany, was organised and conducted as a part of the scientific programme TP-A13 ‘Cyclic and transient wave-induced loading of the seabed’ (in German: ‘Transiente und zyklische Beanspruchung von Böden durch Seegang’) within the frame of the Coastal Engineering Research Programme (in German: Sonderforschungsbereich – 205 ‘Küsteningenieurwesen’). The main purpose of the expedition was to collect many samples of the upper part of the seabed sandy sediments by performing in-situ sampling followed by laboratory measurements in order to evaluate the degree of saturation existing under the natural environmental conditions of the tidal region of Norderney.

Norderney (Figure 1) is one of the seven populated East Frisian Islands and part of the Lower Saxony Land

(state). The island is 13 km long and up to 2 km wide, with dunes rising to 21 metres. The northern coast is exposed to the sea. During low tide, the beach width can reach 550 m. The beach comprises well-sorted quartz fine sand with a median grain size of $d_{50} = 218 \mu\text{m}$ (Eichmanns and Schüttrumpf, 2021). Norderney has a mean tidal range of approximately 2.4 m, which stays within the mesotidal range. The annual mean significant wave height varies from 0.7 m to 1.0 m (Niemeyer, 1992).

A particular place chosen for the seabed sampling is marked in Figure 1. The sampling site was located on the northern beach of Norderney. The sampling was preceded by geodetic measurements, as a result of which a vertical cross-section of the beach along the sampling line was obtained (Figure 2). The zone of seabed sampling was located within the tidal (intertidal) range, i.e., between the High Tide (HT) and the Low Tide (LT) levels of sea water tidal movements. The extreme locations of the sampling points, obtained with respect to the vertical datum of the Mean Sea Level (MSL), had the following abscissas and elevations: $x = 70 \text{ m}$ and $y = 1.21 \text{ m}$ for Point 2 and $x = 415 \text{ m}$ and $y = -1.10 \text{ m}$ for Point 30 (see Figure 2). The total length of the sampling zone was equal to 345 m, which implies that the mean intertidal beach slope was rather mild, with the average inclination $m = [1.21 - (-1.10)]/345.0 \cong 0.0067 \cong 1 : 150$.

2.1 Procedure of seabed sampling

During the sampling campaign, 186 samples of sandy seabed sediments were taken underwater during different phases of the tidal water motion (see Figure 2):

- Day 1, series 1, phase of ebbing, sample nos. 1–38 (38 samples).
- Day 2, series 2, phase of flooding, sample nos. 39–78 (40 samples).
- Day 2, series 3, phase of ebbing, sample nos. 79–127 (49 samples).
- Day 3, series 4, phase of flooding, sample nos. 128–186 (59 samples).

The beginning of sampling started approximately at one of the extreme water levels, i.e., either the High Tide



Figure 1. Aerial view of Norderney, indicating the place of seabed sampling (based on Wikipedia.com (2025)).

156 (HT) or the Low Tide (LT), and was performed stepwise
 157 during the ebbing or flooding phases of the tidal movement,
 158 respectively, when the water level was moving towards an-
 159 other extreme position. All the samples were taken when
 160 the seabed surface at the sampling point was below the ac-
 161 tual water level. The water depth at the sampling point was
 162 always kept equal to approximately 0.1–0.2 m. A smaller
 163 depth could cause some losses of water within the sand
 164 sample during its ‘excavation’ from the seabed, and be-
 165 cause of existing small waves, it could not produce per-
 166 manent underwater conditions for the surface of the sam-
 167 pling area. On the other hand, a bigger water depth could
 168 cause difficulties in the sampling procedure, as it would
 169 be more difficult to bring the whole undisturbed (i.e., nat-
 170 urally saturated) sandy sample above the water surface.
 171 The subsequent sampling points followed the water line’s
 172 actual positions.

173 The sampling device used in the procedure consisted of
 174 the following three tools made of aluminium: a sampling
 175 cylinder, a circular plate with a guiding cylinder, and a pis-
 176 ton. A perforated upper surface of the piston protected
 177 against the creation of artificial and disturbing filtration of
 178 seawater through the soil sample when the sampling cylin-
 179 der was slowly pressed down by the piston into the seabed.
 180 After pressing the sampling cylinder into the seabed (made
 181 using the guiding cylinder and the piston) and levelling the
 182 upper surface of the sand sample, the top of the cylinder
 183 was slowly capped. Then, digging out the sample was per-
 184 formed at a slow rate in order to avoid the suction effect
 185 on the sample. At the end, the sample was lifted above the
 186 water level and simultaneously turned upside-down to pre-
 187 serve the natural water content within the sample and to
 188 allow the levelling of the sand surface from the other side
 189 of the sample. Just after extracting from the seabed, the

sampling cylinder was emptied carefully and precisely, and
 single soil samples were stored alone in separate, identical
 plastic and hermetic containers. Afterwards, all the con-
 tainers, together with the wet sand samples inside, were
 weighed one by one, and then, they were dried in a dryer
 (temperature of 105°C applied for 24 hours). At the end
 of the laboratory procedure, the dry sand samples and the
 respective dry containers were weighed.

2.2 Governing equations

Directly, the following parameters were measured
 (weighed) at the laboratory: total mass of wet sample
 (sand sample plus container), $m_{w(s+c)}$ [g], mass of dry
 container, m_{dc} [g], and mass of dry sand sample, m_{ds} [g].
 Consequently, it became possible to measure the mass of
 the wet soil sample, m_{ws} [g], and the mass of water in the
 sample, m_w [g]:

$$m_{ws} = m_{w(s+c)} - m_{dc} = \rho_{ws} V_t \quad (1a)$$

$$m_w = m_{ws} - m_{ds} \quad (1b)$$

and the rest of the required parameters:

$$V_w = \frac{m_w}{\rho_{sw}} \quad (1c)$$

$$V_s = \frac{m_{ds}}{G_s \rho_w} \quad (1d)$$

$$V_v = V_t - V_s \quad (1e)$$

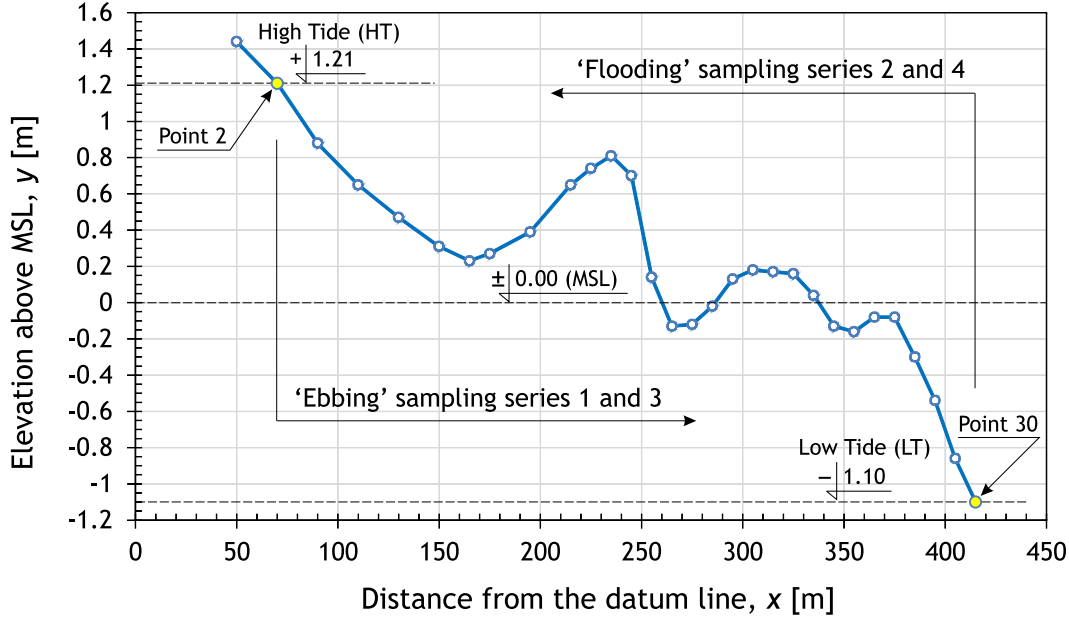


Figure 2. Vertical profile of Norderney beach in the sampling line within the intertidal range of water surface movement.

where, additionally: V_w is the volume of water in the sample [cm³], V_s is the volume of solid soil particles [cm³], V_v is the volume of voids in the soil skeleton [cm³], V_t is the total volume of the soil sample [cm³], G_s is the specific gravity of the soil particles (defined with respect to the density of pure water, ρ_w , at temperature 20°C) [-], ρ_{ws} is the density of the wet sample [g/cm³], ρ_w is the density of pure water [g/cm³], ρ_{sw} is the density of seawater [g/cm³].

Using the basic definition of the degree of saturation (Craig, 2004)

$$S_r \stackrel{\text{def}}{=} \frac{V_w}{V_v} \quad (2)$$

taking Eqs. (1a)–(1e) together with Eq. (2), and performing some mathematical operations, it was possible to derive the following practical form of the equation for the degree of saturation

$$S_r = \frac{m_{w(s+c)} - m_{dc} - m_{ds}}{\rho_w V_t - \frac{m_{ds}}{G_s}} \times \frac{\rho_w}{\rho_{ws}} \quad (3)$$

As indicated in Eq. (3), five measured parameters ($m_{w(s+c)}$, m_{dc} , m_{ds} , G_s , and V_t) must be known to calculate the degree of saturation, S_r . Each weighing carried out with respect to $m_{w(s+c)}$ and m_{ds} was performed with the instrument accuracy (weighing precision or readability) of $\Delta m_{w(s+c)} = \Delta m_{ds} = 0.5$ g (a scale balance – accuracy is equal to the smallest increment divided by 2), whereas the instrument accuracy for measurements of m_{dc} was

equal to $\Delta m_{dc} = 0.1$ g (a digital display electronic balance – accuracy is equal to the smallest increment).

The specific gravity of soil was measured using the pycnometer technique and calculated according to the following relation

$$G_s = \frac{m_2 - m_1}{(m_4 - m_1) - (m_3 - m_2)} \quad (4)$$

where: G_s is the specific gravity of soil solids [-], m_1 is the mass of the pycnometer [g], m_2 is the mass of the pycnometer plus soil [g], m_3 is the mass of the pycnometer plus soil plus pure (distilled) water [g], m_4 is the mass of the pycnometer plus pure (distilled) water [g]. The specific gravity of the soil particles at 20°C was found to be $G_s = 2.654$ (typical for quartz-type sand), calculated as the mean from 8 samples randomly selected (2 samples per each of 4 series) from the whole group of 186 seabed samples. The laboratory measurements of G_s were performed using the pycnometer bottle technique, and the component masses were measured with the instrument accuracy equal to $\Delta m_1 = \Delta m_2 = \Delta m_3 = \Delta m_4 = 0.1$ g (a digital display electronic balance). According to the geotechnical practice, values of G_s are usually rounded to the nearest 0.01 (rounding to the nearest 0.001 is used in the present study).

The total volume of the sample is the same as the volume of the sampling cylinder, which was calculated using the following simple equation

$$V_t \equiv V_c = \frac{\pi D_t^2}{4} h \quad (5)$$

where: V_t is the total volume of the seabed sample [cm³], V_c is the volume of the sampling cylinder [cm³], D_i is the inside diameter of the sampling cylinder [cm], h is the height of the sampling cylinder [cm]. The geometry of the sampling cylinder was measured only once, leading to the following dimensions: the outside diameter $D_o = 9.85$ cm, the inside diameter $D_i = 9.6$ cm, and the height $h = 12.0$ cm. These parameters imply the total volume of samples is equal to $V_t \equiv V_c = 868.59$ cm³. The measurements of the cylinder geometry were performed using a digital caliper with the instrument accuracy equal to $\Delta D_i = \Delta h = 0.01$ cm. The disturbance surface ratio, which is the ratio of the material cross-sectional area to the outer cross-sectional area of the sampling cylinder, was equal to 5%, which is fully acceptable for that type of test, as far as possible meaningful disturbances during the cylinder pressing are taken into account.

The density of water, influenced by temperature, pressure, and salinity, was computed using very precise formulas presented by Gill (1982) and Kell (1975). The density of pure water, at the standard pressure $p_{at} = 101.325$ kPa and the standard temperature $t = 20^\circ\text{C}$ (applied for pycnometer measurements), was equal to $\rho_w = 998.21$ kg/m³. The seawater temperature during the seabed sampling equaled $t = 6.8^\circ\text{C}$, whereas the practical salinity of seawater, typical for the northern coastline of Norderney, is reported to be $S_p = 30$ with only a very small seasonal variability (Quante and Colijn, 2016). Taking it into account, the density of seawater was calculated to be $\rho_{sw} = 1023.51$ kg/m³.

3. Statistical analysis of the measurements

3.1 Basic descriptive statistics

On the basis of the values measured in the laboratory and calculated for each seabed sample taken on Norderney, further data analysis was performed using a statistical approach, by applying the following four commonly accepted basic statistical parameters of the random variable \mathbf{x} defined with respect to a sample taken from the population: the mean (average), \bar{x} , the median, \tilde{x} , the standard deviation, $s(x)$, and the standard deviation of the mean, $s(\bar{x}) = s(x)/\sqrt{n}$ (n is the number of repeated measurements in the set). The descriptive statistics, applied to the measured and calculated parameters, obtained for the total group of 186 seabed samples, are presented in Table 3.

3.2 Histograms

A histogram is a visual representation of the distribution of quantitative data. The total area of a histogram used for probability density is usually normalised to 1. A normalised number of samples can be calculated according to the following recipe

$$n_N = \frac{n_B s(x)}{n B} \quad (6)$$

Table 3. Descriptive statistics for the basic parameters.

Parameter	Unit	Mean \bar{x}	Median \tilde{x}	Standard deviation $s(x)$	Standard deviation of the mean $s(\bar{x})$
$m_{w(s+c)}$	[g]	1906.84	1908.0	23.54	1.73
m_{dc}	[g]	124.16	123.45	1.36	0.10
m_{ws} (Eq. (1a))	[g]	1782.67	1784.0	23.26	1.71
m_{ds}	[g]	1472.09	1472.0	22.78	1.67
G_s	[-]	2.654	2.654	0.008	0.003
S_r (Eq. (3))	[-]	0.973	0.974	0.045	0.003

where: n_N is the normalised number of samples in a histogram bin, n_B is the number of samples in a histogram bin, n is the total number of samples, $s(x)$ is the standard deviation of a real-valued random variable \mathbf{x} , B is the width of a histogram bin. In constructing histograms, for $n > 100$, it usually works well to use a maximum of about $10 \times \log_{10} n$ bins (Fox, 2016). In the present case of 186 samples, the condition $n = 186 > 100$ indicates that the number of bins should not exceed $10 \times \log_{10} 186 = 22.67 \approx 22$. In the case of the mass of the total wet sample, $m_{w(s+c)}$, the minimum and the maximum were equal to 1825.0 g and 1974.0 g, respectively. Covering the entire range from 1820.0 g to 1980.0 g with bins of the width $B = 10$ g, the total number of $n_B = 16$ bins is required. In the case of the degree of saturation, S_r , starting the histogram distribution from $S_r = 0.84$ and ending with $S_r = 1.12$, all the data were represented, including both the minimum and maximum values, $S_r = 0.848$ and $S_r = 1.112$, respectively. The range $S_r = 0.84 - 1.12$, together with the assumed bin width $B = 0.02$, implies $n_B = (1.12 - 0.84)/0.02 = 14$ bins. Both numbers of bins fulfil the above-given condition ($n_B \leq 22$).

Taking into account the initially measured parameter, which is the mass of the total wet sample (wet soil plus container), $m_{w(s+c)}$, and the resulting calculated parameter, which is the degree of saturation, S_r , Figures 3 and 4 show the normalised histograms (highlighted by the green bins), of both parameters, respectively. The numbers placed at the bottom of each bin indicate the number of samples counted for a specific bin. As expected, according to visual assessment, the nomogram tends to follow the normal (Gaussian) probability distribution – this will be proved more precisely in the next paragraph, using formal normality tests. Moreover, the histogram of the degree of saturation, presented in Figure 4, seems to exhibit small asymmetries around its mean. This can be checked by evaluating skewness, which is the distribution's tendency to 'lean' to one side or the other of the mean. In statistics and probability theory, the non-parametric skew is a statistic occasionally used with random variables that take real values – it is a measure of the skewness of a random variable's distribution. Taking the histogram of the degree of saturation (see Figure 4), and the appropriate values given in

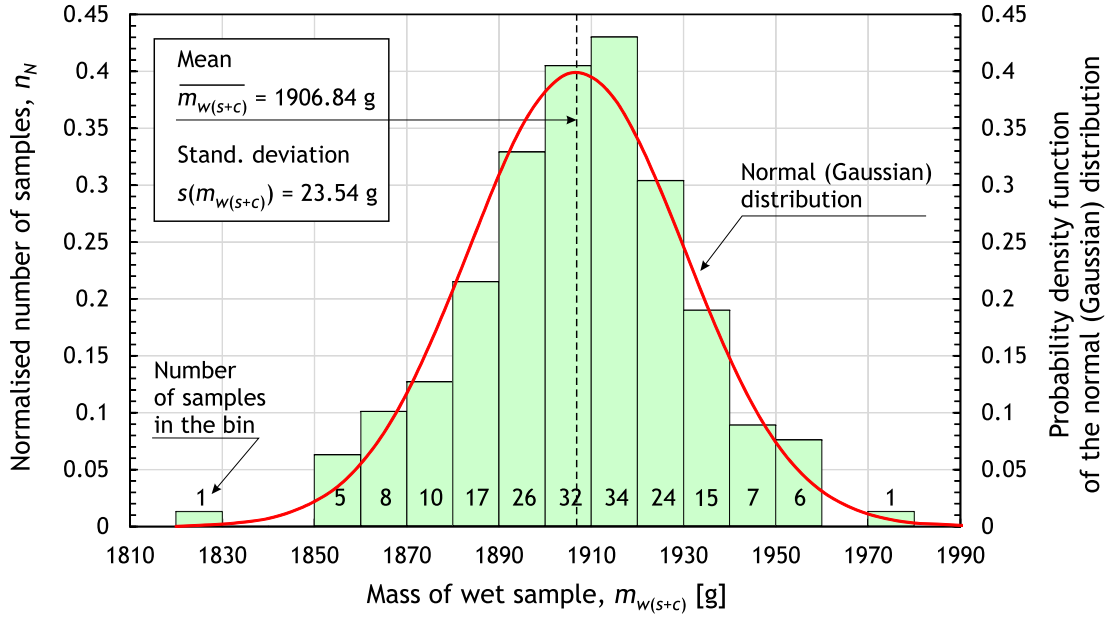


Figure 3. Normalised histogram of the mass of total wet sample (wet soil plus container), $m_{w(s+c)}$, overlaid by the normal (Gaussian) distribution of probability.

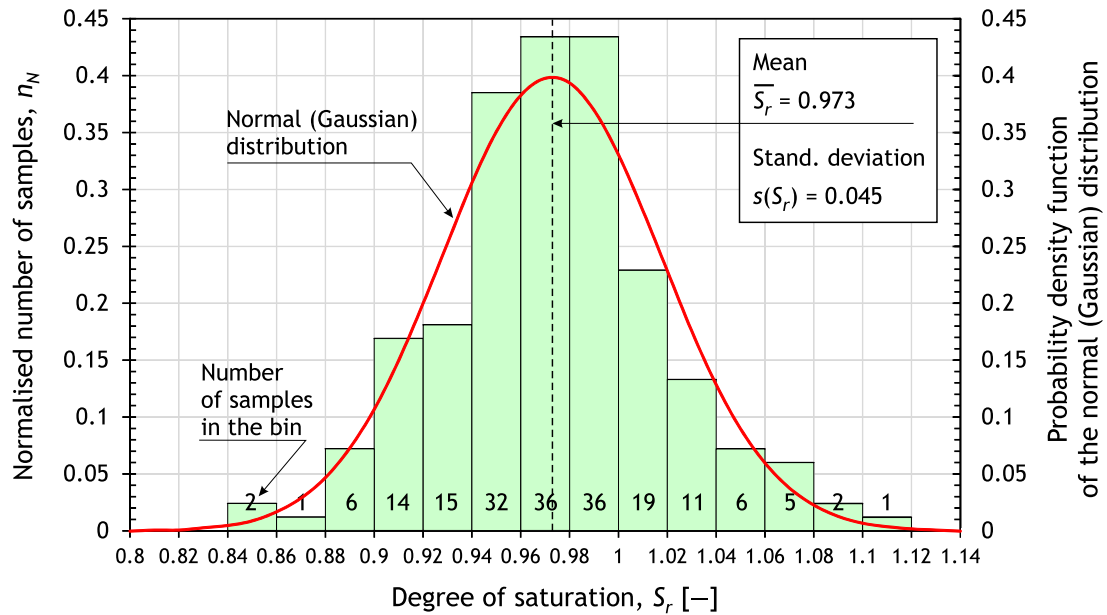


Figure 4. Normalised histogram of the degree of saturation, S_r , overlaid by the normal (Gaussian) distribution of probability.

346 Table 3, the non-parametric skew equals

$$S(S_r) = \frac{\bar{S}_r - \tilde{S}_r}{s(S_r)} = \frac{0.973 - 0.974}{0.045} = -0.022 \quad (7)$$

347 Therefore, this type of distribution is said to be left-
348 skewed and the distribution appears as a right-leaning

curve, which means that the left tail of the distribution is longer than the right one (the mass of the distribution is concentrated on the right of the figure).

3.3 Tests on data normality

In statistics, normality tests are used to determine if a data set is well modelled by a normal distribution. In other

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words, they check if the shape of the distribution under consideration is similar to the shape of the normal (Gaussian) distribution. One of the most powerful tests for normality is the Shapiro-Wilk (S-W) test and the d'Agostino-Pearson test. The so-called null hypothesis, H_0 , of these tests is that the population is normally distributed. If the p -value is greater than a certain predefined significance level, then the null hypothesis is not rejected. The significance level is commonly set a priori to $\alpha = 0.05$.

The modification of the Shapiro-Wilk test, proposed by Royston (1982) for a large number of data ($n = 2000$), was used in the present work. In order to perform the S-W test, the program codes DERF.for and DIERFC.for (written in Fortran 77 by Ooura (1996)) were adapted to compute the error function and the inverse complementary error function, respectively. The main results of the normality tests are presented in Table 4.

For the random variable represented by the degree of saturation, S_r , the following test results were obtained: the test statistic $W = 0.988$ and the probability value $p = 0.134$. The probability value fulfils the condition $p > \alpha$; therefore, the null hypothesis can be accepted with only a 5% risk of a false positive conclusion. It means that there is evidence that the degree of saturation data tested can be treated as normally distributed. This finding was additionally proved by using another powerful normality test, that is the d'Agostino-Pearson test, from which the following test results were obtained: the test statistic $\chi^2 = 3.612$ and the probability value $p = 0.164$.

Unfortunately, the distribution of the mass of dry container, m_{dc} , failed the normality test (Shapiro-Wilk test: $W = 0.825$ and $p = 1.1 \times 10^{-13} \cong 0 < 0.05$; d'Agostino-Pearson test: $\chi^2 = 23.56$ and $p = 7.7 \times 10^{-6} \cong 0 < 0.05$). Firstly, this is due to meaningfully different masses of the sample containers, although they were believed to be identical. Secondly, the independence of the masses was corrupted entirely by accident and unknowingly, using the same set of containers for the wet sand samples nos. 129–186 as for the samples nos. 1–58. It obviously makes no sense to average the significantly different masses m_{dc} because they are not different measurements of the same quantity. It means that due to these mistakes, a statistical analysis cannot be applied to the direct measurements of m_{dc} but can lead to the final answer. Therefore, it seemed wise to replace the term $m_{w(s+c)} - m_{dc}$ in Eq. (3) by the term m_{ws} , according to Eq. (1a). This operation was possible and fully justified because the mass of the wet soil sample, m_{ws} , appeared to be normally distributed, which was certified by performing the normality tests (Shapiro-Wilk test: $W = 0.993$ and $p = 0.513 > 0.05$; d'Agostino-Pearson test: $\chi^2 = 3.034$ and $p = 0.219 > 0.05$).

The p -values presented in Table 4 indicate a partial degradation (up to a certain acceptable level of $p > \alpha = 0.05$) in normality of the distributions, starting from m_{ws} distribution, through m_{ds} distribution, and ending up with

S_r distribution, due to some subsequent steps in measuring and calculation procedures. It has to be also mentioned that the above-presented results of the normality tests take into account the existence of outliers in the individual datasets.

3.4 Probability density distribution and Q-Q plot

In statistics, the standard score or z -score is the number of standard deviations by which the value of a raw score (i.e., an observed or measured value) is above or below the mean value of what is being observed or measured. When the population mean and the population standard deviation are unknown, the standard score may be estimated by using the sample mean and sample standard deviation as estimates of the population values. In these cases, a raw score x is converted into the z -score by

$$z = \frac{x - \bar{x}}{s(x)} \quad (8)$$

It is a very common practice that the variability in a certain real-valued soil parameter, presented in the form of a histogram graphic, is compared assuming the validity of the standard normal (Gaussian) distribution, for which the probability density function is defined as (Bendat and Piersol, 1971)

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-z^2}{2}\right) \quad (9)$$

Respectively of the normalised histogram, it is also typical for the probability density function, given by Eq. (9), that the area under the normal distribution curve with respect to the horizontal axis equals unity. The application of Eq. (9) to the degree of saturation data leads to the normal distribution curve overlaid on the formerly established histogram (see Figure 4). A good agreement between the histogram and the theoretical curve of the normal distribution of probability is observed.

In statistics, a quantile-quantile plot, or shortly – Q-Q plot, is a graphical method commonly used to help in assessing if a set of data plausibly compares to a certain theoretical model of probability distribution. It works by comparing the quantiles of the observed (measured) data to the quantiles of a theoretical distribution. In the case of the normal (Gaussian) distribution specifically, it is usually referred to as a normal Q-Q plot. Thus, in addition to the histogram presented in Figure 4, Figure 5 illustrates the normal Q-Q plot for the degree of saturation data. The ordinate axis in the Q-Q plot presents the z -score (sample data quantiles), calculated for each value of the degree of saturation according to Eq. (8), and the abscissa presents the z_p -score (normal theoretical quantiles), computed from the empirical probability obtained for the rank of the S_r values sorted in descending order and using the

Table 4. Test statistics and probability values obtained from the statistical tests on normality of the datasets representing distributions of the parameters appearing in Eq. (3).

Parameter	Shapiro-Wilk test		d'Agostino-Pearson test		$p \geq \alpha = 0.005$
	W	p -value	χ^2	p -value	
$m_{w(s+c)}$	0.990	0.220	1.758	0.415	Yes
m_{dc}	0.825	1.1×10^{-13}	23.56	7.7×10^{-6}	No
m_{ws} (Eq. (1a))	0.993	0.513	3.034	0.219	Yes
m_{ds}	0.992	0.373	2.174	0.337	Yes
G_s	0.921	0.487	0.993	0.609	Yes
S_r (Eq. (3))	0.988	0.134	3.612	0.164	Yes

inverse empirical cumulative distribution function (eCDF) to find the quantiles of the empirical distribution. The computation of the z_p -score requires the use of the probit function, which is the quantile function (the inverse of the cumulative distribution function (CDF)) associated with the standard normal distribution. The probit function can be expressed as

$$\begin{aligned} \text{probit}(p) &\stackrel{\text{def}}{=} \phi^{-1}(p) = z_p = \\ &= \sqrt{2} \operatorname{erfc}^{-1}[1(1-p)] \end{aligned} \quad (10)$$

where: p is the probability (or percentile in case of the empirical distribution), $\phi^{-1}(p)$ is the inverse CDF, and erfc^{-1} is the inverse complementary error function. For the purpose of the present study, the program code DIERFC.for (Ooura, 1996) was used again to compute the inverse complementary error function.

A linear regression analysis led to the determination of the best-fit trend line (red colour in Figure 5) in the Q-Q plot. The linear regression function was described by the slope $a = 0.9949$ and the z -intercept $b = -0.1418 \times 10^{-5}$. The coefficient of determination was found to be $R^2 = 0.9882$, which is very close to unity. Somewhat approximate linearity of the points in Figure 5 suggests that the degree of saturation data can be described as normally distributed. Only a few outliers are evident at the high end of the range.

Figure 5 also presents two different intervals closed between their upper and lower bands. The 95% confidence interval (bands in blue) relates to a 95% probability that the true best-fit (regression) line for the population lies within this interval. The 95% prediction interval (bands in green) illustrates the area where 95% of the z values to be found for a certain z_p value will be within the interval range around the linear regression line. In other words, if one ever uses another sample from the same population, one might find 95% z values within the prediction interval.

The vertical coordinates of the confidence bands, z_{conf} , and the prediction bands, z_{pred} , can be calculated for the horizontal coordinate, z_p , using the following equations:

$$\begin{aligned} z_{conf} &= \\ &= z_{reg} \pm t_{1-\alpha/2, n-2} SE \sqrt{\frac{1}{n} + \frac{(z_p - \bar{z}_p)^2}{SS}} \end{aligned} \quad (11a)$$

$$\begin{aligned} z_{pred} &= \\ &= z_{reg} \pm t_{1-\alpha/2, n-2} SE \sqrt{1 + \frac{1}{n} + \frac{(z_p - \bar{z}_p)^2}{SS}} \end{aligned} \quad (11b)$$

where: z_{reg} is the vertical coordinate of the linear regression line, $t_{1-\alpha/2, n-2}$ is the t -value obtained from the inverse cumulative two-tailed Student's t -distribution function for the assumed significance level α , SS is the sum of squared deviations, and SE is the standard error for the sample mean.

The significance level $\alpha = 0.05$ is a typical value assumed when constructing the confidence and prediction intervals. The t -value can be found from a t -distribution table. It can also be calculated using the significance level, α , and the number of degrees of freedom. The significance level, α , is divided by two for a two-tailed test, so the area in each tail is $\alpha/2$. In the present statistical analysis of the degree of saturation data, the t -value was equal to $t = 1.9729$ for the number of degrees of freedom $n_{df} = n - 2 = 184$ and the significance level $\alpha = 0.05$ ($t_{1-\alpha/2} \equiv t_{0.975}$). For a large number of degrees of freedom (e.g., $n_{df} > 30$), as it is in the present study, the critical t -value can also be safely approximated by the limiting z value obtained from the inverse normal cumulative distribution function (CDF); here $t \cong z = 1.96$.

3.5 Identification of outliers

Outliers are defined as numeric values in any random data set that have an unusually high deviation from either the statistical mean or the median value. Four different methods of locating outliers were used: a) z -score method, b) Median Absolute Deviation (MAD) method, c) Tukey's Interquartile Range (IQR) method, and d) Chauvenet's criterion.

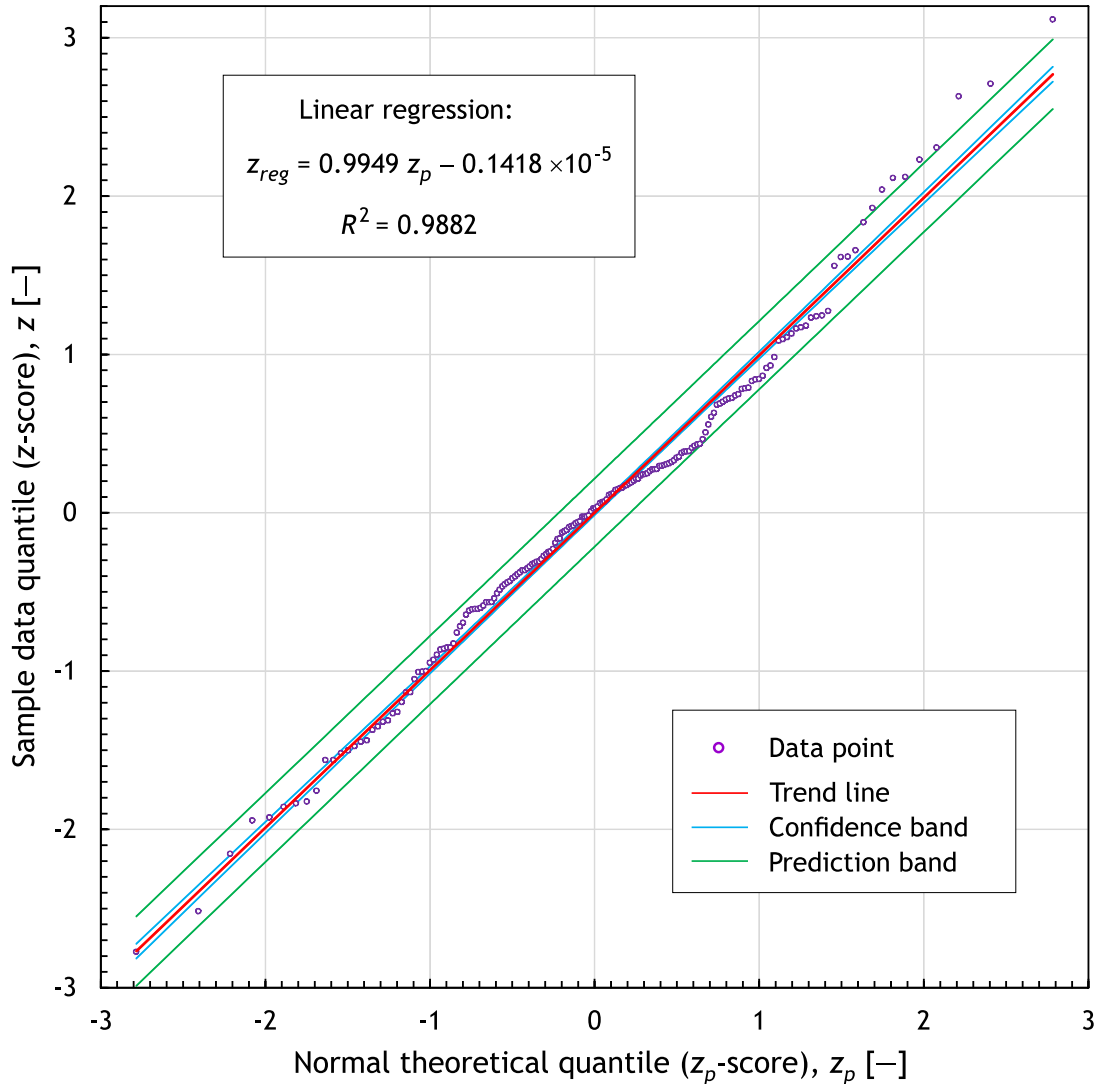


Figure 5. Normal quantile-quantile (Q-Q) plot for the degree of saturation data together with the best-fitted trend line and the confidence and prediction intervals.

521 The first method is rather simple. When the z-score of
 522 an observation point has a probability under 5% (confi-
 523 dence lever 95%, or 5% error chance), one could consider
 524 that observation an outlier. The second method is a more
 525 robust statistical technique based on the Median Absolute
 526 Deviation (MAD). Any number in a data set exceeding 3.5
 527 times MAD is considered an outlier. In the third method,
 528 Tukey called any observation value an outlier if it is smaller
 529 than the first quartile (25%) minus 1.5 times the IQR, or
 530 larger than the third quartile (75%) plus 1.5 times the IQR.
 531 The Interquartile Range (IQR) is the width of the interval
 532 that contains the middle half of the data. Tukey's method
 533 is probably the best one in assessing outliers. The fourth
 534 method, based on Chauvenet's criterion, detects outliers
 535 by computing the absolute value of the difference between
 536 the suspected outliers and the mean divided by the sample

standard deviation and comparing it with the maximum
 allowable deviation, which is obtained from the quantile
 function for the probability $p = 1 - 1/(4n)$.

537 Deletion of outlier data, although possible to be per-
 538 formed, is rather a controversial practice frowned upon by
 539 many scientists. Table 5 presents the number of outliers
 540 identified by the above-given methods together with the
 541 associated mean values of the degree of saturation, \bar{S}_r , cal-
 542 culated after exclusion of the outliers from the initial set of
 543 the calculated S_r values. The corrected \bar{S}_r values show very
 544 insignificant differences in relation to the mean calculated
 545 for the entire set of data ($\bar{S}_r = 0.973$). The differences
 546 vary from -0.31% for the z-score method to -0.10% for
 547 Chauvenet's criterion.
 548
 549
 550

551 Removing the identified outliers from a dataset obvi-
 552 ously improves the normality of the distribution. For exam-

Table 5. Number of potential outliers detected by different methods.

Method of identification of outliers	Number of outliers n_{out}	Percentage of outliers n_{out}/n [%]	Corrected mean of S_r $(\overline{S_r})_{cor}$
z-score	11	5.91	0.970
Median Absolute Deviation	10	5.38	0.970
Tukey's Interquartile Range	7	3.76	0.971
Chauvenet's criterion	1	0.54	0.972

553 ple, eliminating 7 outliers from the dataset of the degree of
554 saturation (as indicated by Tukey's method), the p -value in-
555 creases from $p = 0.134$ (7 outliers included in the dataset)
556 to $p = 0.338$ (7 outliers excluded from the dataset).

557 3.6 Propagation of uncertainties

558 Computing uncertainty in parameters, which are based on
559 more complicated functions, can be done using the Law
560 of Propagation of Uncertainty (Root Sum of Squares (RSS)
561 method) for independent random uncertainties. After as-
562 suming independence and randomness of the variables in
563 Eq. (3), together with the above-introduced replacement
564 of the masses, one can use the first two terms of the Taylor
565 series to obtain a first-order approximation of the variabil-
566 ity in the degree of saturation. Assuming that the specific
567 densities of pure water, ρ_w , and seawater, ρ_{sw} , are constant,
568 and using the RSS method for uncertainty in a function of
569 several variables, the following gives the differential equa-
570 tion for the total (complete) standard uncertainty in the
571 degree of saturation

$$572 \quad u_t(S_r) \cong \sqrt{\left[\frac{\partial S_r}{\partial m_{ws}} u_c(m_{ws})\right]^2 + \left[\frac{\partial S_r}{\partial m_{ds}} u_c(m_{ds})\right]^2 + \left[\frac{\partial S_r}{\partial V_t} u_c(V_t)\right]^2 + \left[\frac{\partial S_r}{\partial G_s} u_c(G_s)\right]^2} \quad (12)$$

573 where the symbol $u_c(\dots)$, together with a certain parame-
574 ter symbol enclosed in round brackets, denotes the com-
575 bined standard uncertainty in this parameter and can be
576 calculated as a combination of the random (statistical)
577 standard uncertainty (from repeated measurements), u_r ,
578 and the instrument-based (systematic) standard uncer-
579 tainty (e.g., from instrument calibration, resolution, etc.),
580 u_i , according to the following combination in quadrature

$$581 \quad u_c(\dots) = \sqrt{u_r^2(\dots) + u_i^2(\dots)} \quad (13)$$

581 Using the corrected Eq. (3), i.e. with m_{ws} instead of
582 $m_{w(s+c)} - m_{dc}$, and applying the quotient rule of differen-
583 tiation, it becomes possible to derive individual parts of

Eq. (12). The final forms of the derivatives, appearing in
Eq. (12), are presented in the Appendix section (see Eqs.
(A-1)–(A-4)).

When reporting the average value of n measurements,
the uncertainty associated with this average value is the
standard deviation of the mean, often called the standard
error (SE). In order to convert an instrument's maximum
uncertainty range into standard uncertainty, one has to
divide the half-range by $\sqrt{3}$, assuming that a rectangular
distribution of the instrument's uncertainty is valid. There-
fore, for the present study, one obtains (see Table 3):

$$584 \quad u_c(m_{ws}) = \sqrt{u_r^2(m_{ws}) + u_i^2(m_{w(s+c)}) + u_i^2(m_{ws})} = \sqrt{s^2(\overline{m_{ws}}) + \left(\frac{\Delta m_{w(s+c)}}{2\sqrt{3}}\right)^2 + \left(\frac{\Delta m_{ws}}{2\sqrt{3}}\right)^2} = 1.71^2 + \left(\frac{0.5}{\sqrt{3}}\right)^2 + \left(\frac{0.5}{\sqrt{3}}\right)^2 = 1.76 \text{ g} \cong 2 \text{ g} \quad (14a)$$

$$585 \quad u_c(m_{ds}) = \sqrt{u_r^2(m_{ds}) + u_i^2(m_{ds})} = \sqrt{s^2(\overline{m_{ds}}) + \left(\frac{\Delta m_{ds}}{2\sqrt{3}}\right)^2} = 1.67^2 + \left(\frac{0.5}{\sqrt{3}}\right)^2 = 1.69 \text{ g} \cong 2 \text{ g} \quad (14b)$$

586 Please note that the above-given uncertainties were
587 rounded to one significant digit, according to the rule for
588 stating uncertainties.

589 Propagation of uncertainty for several variables can
590 be simplified considerably for the special case where the
591 function is a simple multiplicative function of secondary
592 variables and uncertainty is evaluated as a percentage (the
593 so-called fractional uncertainty). Thus, taking the case of
594 the total volume of the sample, V_t (see Eq. (5)), and using
595 the rules for uncertainty in a measured quantity times an
596 exact number and in a power, the instrument's uncertainty
597 in V_t can be calculated from the following equation

$$598 \quad \frac{\Delta V_t}{V_t} = \sqrt{\left(2 \frac{\Delta D_i}{D_i}\right)^2 + \left(\frac{\Delta h}{h}\right)^2} \quad (15)$$

599 Taking into account the formerly mentioned compo-
600 nent instrument uncertainties ($\Delta D_i = \Delta h = 0.01 \text{ cm}$), the
601 single measured dimensions of the sampling cylinder ($D_i =$
602 9.6 cm and $h = 12 \text{ cm}$), and the calculated volume $V_t =$
603 9.6 cm and $h = 12 \text{ cm}$), and the calculated volume $V_t =$
604 9.6 cm and $h = 12 \text{ cm}$), and the calculated volume $V_t =$
605 9.6 cm and $h = 12 \text{ cm}$), and the calculated volume $V_t =$
606 9.6 cm and $h = 12 \text{ cm}$), and the calculated volume $V_t =$

611 868.59 cm³, one has $\Delta V_t/V_t = 0.0022$, and finally the in-
 612 strument's uncertainty $u_i(V_t) = \Delta V_t = 1.95 \cong 2$ cm³ was
 613 obtained from Eq. (15). The random uncertainty in this
 614 single-measured parameter does not apply. Therefore, it
 615 implies that

$$u_c(V_t) = u_i(V_t) = \Delta V_t = 2 \text{ cm}^3 \quad (16)$$

616 In the case of the specific gravity of soil particles, G_s (see
 617 Eq. (4)), the instrument's uncertainty can also be calcu-
 618 lated according to the RSS differential equation. Using Eq.
 619 (4), deriving appropriate derivatives, and taking the labo-
 620 ratory measurement results into account, the combined
 621 standard uncertainty was found to be $u_c(G_s) = 0.004$.

622 Finally, the total standard uncertainty in the degree
 623 of saturation can be calculated from Eq. (12). Taking the
 624 mean (best-estimated) values (see Table 3) and the respec-
 625 tive combined standard uncertainties: $\overline{m_{ws}} = 1782.67$ g
 626 and $u_c(m_{ws}) = 2$ g, $\overline{m_{ds}} = 1472.09$ g and $u_c(m_{ds}) = 2$ g,
 627 $\overline{G_s} = 2.654$ and $u_c(G_s) = 0.004$, together with $V_t = 868.59$
 628 cm³ and $u_c(V_t) = 2.0$ cm³, the total standard uncertainty
 629 in $\overline{S_r}$ was found to be $u_t(S_r) = 0.01$. It implies that one can
 630 reasonably expect (with about 68% confidence; the cover-
 631 age factor is equal to 1) that the mean value of the degree
 632 of saturation will be within the range $\overline{S_r} = 0.973 \pm 0.01$,
 633 which can also be written as

$$0.963 \leq \overline{S_r} \leq 0.983 \quad (17)$$

634 3.7 Influence of the annual variability of seawater con- 635 ditions

636 The above analysis was performed, assuming the envi-
 637 ronmental seawater conditions typical for the northern
 638 coastline of Norderney during the seabed measuring cam-
 639 paign. These conditions were characterised by the seawater
 640 temperature $t = 6.8^\circ\text{C}$ and the practical salinity $S_p = 30$.
 641 The resultant density of seawater was calculated to be
 642 $\rho_{sw} = 1023.51$ kg/m³. However, if one wants to spread
 643 the results of the degree of saturation over the entire an-
 644 nual period, it is necessary to take into account the annual
 645 mean and the seasonal variability of the sea surface tem-
 646 perature and salinity on the northern coast of Norderney.
 647 Based on the data from the period 1900–1996, the follow-
 648 ing values were found: $\overline{t} = 10.5^\circ\text{C}$ with $\Delta t = 7.5^\circ\text{C}$, and
 649 $\overline{S_p} = 30$ with $\Delta S_p = 1$ (Quante and Colijn, 2016).

650 Using the practical formulas by Gill (1982) and Kell
 651 (1975) again, the extreme values of the density of water,
 652 influenced by temperature, pressure (the standard pres-
 653 sure $p_{at} = 101.325$ kPa was assumed), and salinity, were
 654 calculated, and they are as follows:

$$(\rho_{sw})_{\min} = 1020.68 \text{ kg/m}^3 \text{ at } t = 18^\circ\text{C} \quad (18a)$$

and $S_p = 29$

$$(\rho_{sw})_{\max} = 1024.69 \text{ kg/m}^3 \text{ at } t = 3^\circ\text{C} \quad (18b)$$

and $S_p = 31$

656 Accordingly, the following correction factors can be
 657 calculated:

$$(c_\rho)_{\min} = \frac{\rho_{sw}}{(\rho_{sw})_{\max}} = \frac{1023.51}{1024.69} = 0.999 \quad (19a)$$

$$(c_\rho)_{\max} = \frac{\rho_{sw}}{(\rho_{sw})_{\min}} = \frac{1023.51}{1020.68} = 1.003 \quad (19b)$$

and used by multiplying Eq. (3).

658 It means that the range of possible variations in $\overline{S_r}$,
 659 given in Eq. (17), must be slightly extended (by multiplying
 660 by the correction factors) due to the annual variability of
 661 the seawater temperature and salinity. Consequently, the
 662 lower range is reduced by 0.001 and the upper range is
 663 increased by 0.003, leading finally to the main result of
 664 the possible variability of the degree of saturation of the
 665 northern coastline of Norderney within the intertidal zone
 666 of water level movements

$$0.962 \leq \overline{S_r} \leq 0.986 \quad (20)$$

667 3.8 Tidal cycle difference in the degree of saturation

668 Statistical calculations for the degree of saturation, dis-
 669 tinguishing between the 'ebbing' and 'flooding' groups
 670 of seabed samples taken during the ebbing and flooding
 671 phases of the tidal water movement, respectively, were ex-
 672 ecuted. It can be emphasised that ebbing (dewatering) and
 673 flooding the beach sediments during the low tide and high
 674 tide water movements, respectively, make the degree of
 675 saturation oscillate around the mean value calculated for
 676 all 186 samples. The mean for the ebbing phase (series 1
 677 and 3) was equal to $\overline{S_r} = 0.974$, and for the flooding phase
 678 $\overline{S_r} = 0.972$; the difference is practically negligible.

679 An even more interesting observation is that the spatial
 680 distribution of S_r along the sampling line has an increas-
 681 ing tendency when moving from the high tide (HT) line
 682 towards the low tide (LT) line, irrespective of the series of
 683 sampling (ebbing or flooding). The maximum and mini-
 684 mum differences in S_r were found to be +0.041 (series 1)
 685 and +0.021 (series 3), respectively. This particular find-
 686 ing is confirmed in Figure 6, presenting the results of the
 687 additional spatial analysis obtained in terms of the best-fit
 688 trend lines for the variation of the degree of saturation
 689 along the sampling line. Qualitatively, these results were
 690 expected and they can have a twofold explanation. Firstly,
 691 during the flooding period, the wet seabed sediments en-
 692 trap small amounts of air within the soil voids and, conse-
 693 quently, the degree of saturation is slightly lower than that
 694 obtained during the ebbing phase. Although water filtrates
 695

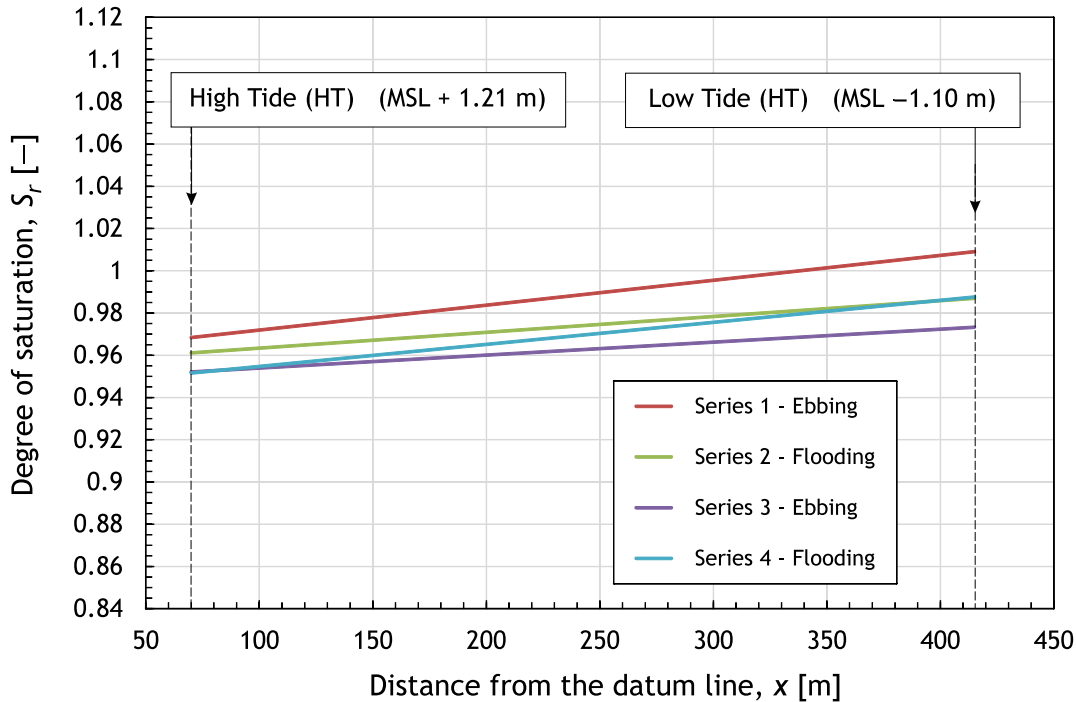


Figure 6. Trend lines for the variation of the degree of saturation, S_r , along the sampling line, calculated for four different measuring series.

696 faster in wet sands than in dry sandy sediments, it seems
 697 that the horizontal velocity of flooding ($345 \text{ m}/6 \text{ h} \cong 0.016$
 698 m/s) evidently dominates the vertical groundwater veloc-
 699 ity ($2.31 \text{ m}/6 \text{ h} \cong 10^{-4} \text{ m}/\text{s}$, which is quite comparable
 700 with the coefficient of permeability of fine sand $k \cong 10^{-4}$
 701 m/s). Secondly, the lower areas of the beach within the
 702 intertidal zone remain below the water surface longer and
 703 are subjected to wave action longer than the upper parts of
 704 the beach. This strongly affects the bottom and causes sig-
 705 nificant oscillatory movements of the bottom sediments,
 706 leading to some release of the air entrapped in the soil
 707 in the form of microscopic occluded bubbles during the
 708 flooding phases of the tidal water movement.

709 In addition to the graphical presentation, statistical
 710 testing by using an unpaired t -test (or independent sam-
 711 ples t -test) seems to be appropriate as a statistical tool
 712 used to compare the means of two separate, unrelated
 713 groups to check if there is a statistically significant dif-
 714 ference between them. In other words, the testing should
 715 determine whether observed differences are due to chance
 716 (random) or a real effect, with results assessed by a cal-
 717 culated statistic t -value and a probability p -value. In the
 718 case of unequal variances, Welch's t -test is widely used and
 719 more reliable than Student's t -test, which is better when
 720 variances are assumed equal. The main assumptions for
 721 the t -test performance are: observations within and be-
 722 tween groups are independent, data in each group should

723 be approximately normally distributed, and the variances
 724 of the two groups should be roughly equal. The input data
 725 required for the t -test are collected in Table 6, and the
 726 results of Welch's t -test are presented in Table 7.

727 Assuming the significant level $\alpha = 0.05$, it can be con-
 728 cluded that the condition $p \geq \alpha = 0.05$ is fulfilled in all the
 729 considered cases. It means that the differences between
 730 the means of the degree of saturation, found for the com-
 731plementary phases (ebbing and flooding) of one full tidal
 732 cycle of the water movement, are not big enough to be sta-
 733tistically significant. The existing small differences could
 734 happen by chance and, therefore, have a random character.

735 3.9 Comments on the degree of saturation above unity 735

736 When reviewing the calculated saturation degrees, it was
 737 found that some of them (44 of 186 tests, or 23.7% of all
 738 tests) exceed unity. From a physical point of view, it is
 739 impossible to have a degree of saturation $S_r > 1$. How-
 740 ever, from an engineering point of view, this result is not
 741 so much surprising because of the nature of the sampling
 742 procedure, the undoubted random variability of measure-
 743 ments, and a certain inhomogeneity of seabed sediments.
 744 It has to be noted that even after introducing the most in-
 745 convenient combination of the instrument's uncertainties
 746 into Eq. (3), lowering thereby the S_r values as much as
 747 possible, there are still 25 tests (15.0% of all tests) remain-
 748 ing with $S_r > 1$. It certifies that both types of uncertainties

Table 6. Statistical values related to the degree of saturation, S_r , and calculated exclusively for the seabed samples taken during the ebbing or flooding phases of the tidal water movement.

Phases of tidal water movement	Number of data n	Mean \bar{S}_r	Standard deviation $s(S_r)$
Series 1 (ebbing)	38	0.989	0.057
Series 2 (flooding)	40	0.974	0.028
Series 3 (ebbing)	49	0.963	0.043
Series 4 (flooding)	59	0.970	0.034

Table 7. Results of Welch's t -test for all possible combinations of the seabed sampling series, reflecting one tidal ebbing-flooding cycle of the water movement.

Merged phases of tidal water movement	Statistic t -value	Probability p -value	$p \geq \alpha = 0.005$?
Series 1 + 2	1.426	0.160	Yes
Series 1 + 4	1.859	0.069	Yes
Series 2 + 3	1.489	0.140	Yes
Series 3 + 4	0.915	0.363	Yes

(random and instrumental), related to sampling disturbances and measurement errors, influence the degree of saturation in exceeding unity. It is also believed that a natural variability (inhomogeneity) of S_r influences the results, but this cannot be clarified quantitatively at the present stage of the research.

According to Schmertmann (1989), rejecting tests with $S_r > 1$ would distort statistical findings from the field sampling and may have important engineering and economic consequences. He performed an excessive statistical analysis of the data gained from the density check conducted at three fills. In his 31 tests (17% from the total number of 187 tests), the degree of saturation was reported to have values above unity ($S_r > 1$), although the mean was equal to $\bar{S}_r = 0.877$. Applying the so-called Zero Air Voids Line (ZAVL) approach, Schmertmann (1989) demonstrated that one should expect a certain percentage of all random tests to compute with $S_r > 1$, that this percentage increases with the mean degree of saturation, \bar{S}_r , and that any such tests have as much validity as the tests with $S_r \leq 1$. After all, it should be clear to everyone that any approximately symmetrical distribution of variability around the mean $\bar{S}_r = 1$ would produce approximately one half of the computed S_r values higher than the mean ($S_r > \bar{S}_r = 1$), and if one eliminates the tests with $S_r > 1$ from the dataset statistical processing, the mean will be artificially corrupted and lower than unity. In the present study, the elimination of 44 values of $S_r > 1$ from the entire dataset would lead to the mean equal to $\bar{S}_r = 0.955$. This action, however, destroys the normality of S_r distribution, which is certi-

fied by the statistic $W = 0.922$ and the probability value $p = 10^{-7} < \alpha = 0.05$ obtained from the Shapiro-Wilk normality test.

From the formerly proven assumption of the normally distributed values of the degree of saturation, it becomes possible to calculate the expected percentage of tests with $S_r > 1$ using the following formula

$$P(S_r > 1.0) = \frac{1}{s(S_r)\sqrt{2\pi}} \int_{1.0}^{\infty} \exp\left\{-\frac{(1.0 - \bar{S}_r)^2}{2[s(S_r)]^2}\right\} dS_r \quad (21)$$

In the present study, this calculation was performed numerically where the procedure of numerical integration, using the extended midpoint rule programmed in subroutine MIDINF (Press et al., 1992), had to be implemented because the integral to be calculated is an improper integral, the upper limit of which approaches infinity (see Eq. (21)). Using Eq. (21), taking the standard deviation $s(S_r) = 0.045$ and the mean $\bar{S}_r = 0.973$ (see Table 3), the analytically evaluated percentage of tests with the degree of saturation exceeding unity was calculated to be 27.4%, compared to 23.7% of values calculated from the field sampling, laboratory measurements, and the use of Eq. (3). This rather small discrepancy can be explained by small differences between the theoretical normal distribution of probability and the experimental normalised distribution of the degree of saturation illustrated in the histogram (see Figure 4).

4. Conclusion

Any information on real saturation conditions of seabed sediments is hard to find in the professional literature. Therefore, a measuring campaign performed on the intertidal region of the northern coast of Norderney created a very unique possibility to obtain realistic characteristics of the degree of saturation. After the in-situ sampling of the seabed on Norderney, followed by appropriate laboratory measurements of 186 sand samples, a thorough statistical analysis of the measured data was performed. Treating the measured soil parameters as real-valued random variables, and using two different normality tests (Shapiro-Wilk test and d'Agostino-Pearson test) together with the histogram and the Q-Q plot, the assumption of the normal (Gaussian) distribution of probability in application to the variability in the degree of saturation, calculated from the measured masses of the seabed samples, was proved. The mean value of the degree of saturation was found to be $\bar{S}_r = 0.973$, whereas a possible range of variation in \bar{S}_r , equal to 0.962–0.986, was calculated based on the analysis of propagation of uncertainties (both random and instrumental), and taking into account the annual variabilities of the seawater temperature and salinity, influencing the seawater density.

This outcome obviously has important consequences for coastal engineering practice. The values of the degree of saturation, obtained from the intertidal range of the seabed, are believed to create the lower limit for S_r values typical for seabeds, which are totally and continuously submerged. On the other hand, it must be strongly emphasised that the obtained mean value $\bar{S}_r = 0.973$ is larger, sometimes much larger, than the values used to this day by many coastal engineers and researchers, thereby putting into question their theoretical results of different analytical and numerical analyses of the wave-induced cyclic seabed response.

Another interesting finding is related to the spatial distribution of the degree of saturation along the sampling line set perpendicular to the coastline. The present research indicated an increasing tendency of S_r distribution when moving from the high tide line towards the low tide line, regardless of the direction of sampling, i.e., during the ebbing or flooding phases of the tidal water movement. By performing Welch's t -test, it was proved that there is no significant difference between the means of the degree of saturation, calculated for the ebbing and flooding phases of the tidal cycle of water movements. The reported changes in S_r within the intertidal zone could be of some engineering interest as far as the design of submarine pipelines buried in seabed sediments is concerned. This is due to a strong influence of the seabed saturation conditions on the pore-fluid pressure field around a buried coastal structure.

Of course, one has to remember that seabed sediments can be characterised by much lower values of S_r than the values obtained from the present study. However, this can happen only due to methane deposits existing along the seabed in deep-water regions, and this exceptional case was of no interest in the present paper.

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Conflict of interest

None declared.

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Appendix

1065
 1066 Partial derivatives of Eq. (3), obtained with respect to the
 1067 component parameters (ρ_w and ρ_{ws} are assumed to be
 1068 constant), are as follows:

$$\frac{\partial S_r}{\partial m_{ws}} = \frac{1}{\rho_w V_t - \frac{m_{ds}}{G_s}} \times \frac{\rho_w}{\rho_{sw}} \quad (\text{A-1})$$

$$\frac{\partial S_r}{\partial m_{ds}} = \frac{-\rho_w V_t + \frac{m_{ws}}{G_s}}{\left(\rho_w V_t - \frac{m_{ds}}{G_s}\right)^2} \times \frac{\rho_w}{\rho_{sw}} \quad (\text{A-2})$$

$$\frac{\partial S_r}{\partial V_t} = \frac{-(m_{ws} - m_{ds})\rho_w}{\left(\rho_w V_t - \frac{m_{ds}}{G_s}\right)^2} \times \frac{\rho_w}{\rho_{sw}} \quad (\text{A-3})$$

$$\frac{\partial S_r}{\partial G_s} = \frac{-(m_{ws} - m_{ds})\frac{m_{ds}}{G_s^2}}{\left(\rho_w V_t - \frac{m_{ds}}{G_s}\right)^2} \times \frac{\rho_w}{\rho_{sw}} \quad (\text{A-4})$$